NATURALLY OCCURRING RADIOACTIVE MATERIAL
(NORM VI)

PROCEEDINGS OF THE
SIXTH INTERNATIONAL SYMPOSIUM ON
NATURALLY OCCURRING RADIOACTIVE MATERIAL
ORGANIZED BY
THE HASSAN II UNIVERSITY OF MOHAMMEDIA AND
THE CADI AYYAD UNIVERSITY OF MARRAKESH
IN COOPERATION WITH
THE INTERNATIONAL ATOMIC ENERGY AGENCY
AND HELD IN MARRAKESH, MOROCCO,
22–26 MARCH 2010

INTERNATIONAL ATOMIC ENERGY AGENCY
VIENNA, 2011
COPYRIGHT NOTICE

All IAEA scientific and technical publications are protected by the terms of the Universal Copyright Convention as adopted in 1952 (Berne) and as revised in 1972 (Paris). The copyright has since been extended by the World Intellectual Property Organization (Geneva) to include electronic and virtual intellectual property. Permission to use whole or parts of texts contained in IAEA publications in printed or electronic form must be obtained and is usually subject to royalty agreements. Proposals for non-commercial reproductions and translations are welcomed and considered on a case-by-case basis. Enquiries should be addressed to the IAEA Publishing Section at:

Marketing and Sales Unit, Publishing Section
International Atomic Energy Agency
Vienna International Centre
PO Box 100
1400 Vienna, Austria
fax: +43 1 2600 29302
tel.: +43 1 2600 22417
email: sales.publications@iaea.org
http://www.iaea.org/books

© IAEA, 2011
Printed by the IAEA in Austria
June 2011
STI/PUB/1497

IAEA Library Cataloguing in Publication Data

Naturally occurring radioactive material (NORM VI): proceedings of the sixth International Symposium on naturally occurring radioactive material organized by the Hassan II University of Mohammedia and the Cadi Ayyad University of Marrakesh in cooperation with the International Atomic Energy Agency and held in Marrakesh, Morocco, 22–26 March 2010. — Vienna : International Atomic Energy Agency, 2011.
   p. : 24 cm. — (IAEA proceedings series, ISSN 0074–1884)
STI/PUB/1497
Includes bibliographical references.


IAEAL 11–00682
A MATHEMATICAL TOOL FOR SIMULATING THE DISPERSION OF NORM RELEASES IN THE MARINE ENVIRONMENT: APPLICATION TO THE WESTERN ALBORAN SEA

A. LAISSAOUI*, R. PERIÁNEZ**, M. BENMANSOUR*

* Centre National de l’Energie, des Sciences et des Techniques Nucléaires (CNESTEN), Rabat, Morocco
Email: laissaoui@cnesten.org.ma

** University of Seville, Dpto. Física Aplicada 1, Seville, Spain.

Abstract

A numerical model which simulates the dispersion of radionuclides of natural origin in the marine environment has been developed and applied to the western Alboran Sea. The model consists of a hydrodynamic module which operates off-line, and the dispersion module itself. The hydrodynamic module consists of two models: a 2D barotropic model which provides instantaneous tidal currents and a 3D baroclinic model which provides the long term residual currents. The calculated currents are stored in files which are later read by the dispersion model. The calculated tidal and residual currents have been compared with measurements in the area. In particular, the well known Western Alboran Gyre is reproduced by the model. The dispersion model essentially solves the advection–diffusion equation using finite difference techniques. Interactions of dissolved radionuclides with suspended particles have been neglected in this case, given the low particle concentrations present in the western Alboran Sea and Strait of Gibraltar. The model has been applied to simulate $^{238}$Ra releases from hypothetical phosphate fertilizer industries located on the Spanish and Moroccan coasts. It is a useful tool for assessing the effects of planned releases of radionuclides of natural origin in the marine environment.

---

1 This work was partially supported by Agencia Española de Cooperación Internacional para el Desarrollo (AECID), PCI Projects A/5066/07 and A/7942/07.
1. INTRODUCTION

The area comprising the Strait of Gibraltar and the Alboran Sea constitutes the connection between the Mediterranean Sea and the Atlantic Ocean and has been the subject of many radioecological studies [1]. The water dynamic is characterized by a surface inflow of Atlantic water and a deep outflow of dense Mediterranean water. The Atlantic jet flows along the Spanish coast and curves to the south to incorporate the so-called Western Alboran Gyre (WAG) [2]. The objective of this work was to construct a dispersion model covering the Strait of Gibraltar and the Alboran Sea that was able to simulate $^{226}\text{Ra}$ releases from hypothetical phosphate fertilizer industries located on the Spanish and Moroccan coasts.

2. MODEL DESCRIPTION

The topography of the area covered by the model can be seen in Fig. 1. The computational domain extends from $35^\circ00'\text{N}$ to $36^\circ74'\text{N}$ and from $05^\circ35'\text{W}$ to $03^\circ00'\text{W}$. Water depths were downloaded from the NOAA Geodas database having a resolution of $\Delta x = 3032$ m and $\Delta y = 3712$ m.

\begin{figure}
\centering
\includegraphics[width=\textwidth]{fig1.png}
\caption{Topography of the area covered by the model.}
\end{figure}
FIG. 2. Tide amplitude (left) and current amplitude (right) for the $M_2$ tide computed by the 2D depth averaged model [3].

2.1. Hydrodynamics

Tidal currents in the area covered by the model were obtained by means of a 2D depth averaged model [3]. The model solves the standard equations and provides the water currents at each compartment of the calculation mesh and at each time step. Currents are treated by standard tidal analysis to obtain tidal constants that are stored in files that will be read by the dispersion code. Tide and current amplitudes computed by the model are presented in Fig. 2 and are in good agreement with measurements and earlier computations.

The 3D hydrodynamic equations are the following [4]:

$$\frac{\partial \xi}{\partial t} + \frac{\partial}{\partial x}(\int u \cdot dx) + \frac{\partial}{\partial y}(\int v \cdot dy) = 0$$

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} - \Omega \cdot v + g \frac{\partial \xi}{\partial z} + \frac{g \xi}{\rho_0} \frac{\partial \rho}{\partial x} \cdot dx$$

$$= \frac{\partial}{\partial z} \left[ K \cdot \frac{\partial u}{\partial z} \right] + A \left[ \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right]$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} - \Omega \cdot u + g \frac{\partial \xi}{\partial y} + \frac{g \xi}{\rho_0} \frac{\partial \rho}{\partial y} \cdot dz$$

$$= \frac{\partial}{\partial z} \left[ K \cdot \frac{\partial v}{\partial z} \right] + A \left[ \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right]$$

(1)
where $\xi$ is the water elevation above the mean level, $g$ is the acceleration due to gravity, $\Omega$ is the Coriolis parameter, $u$ and $v$ are the two components of the velocity, $\rho_w$ is the water density and $K$ and $A$ are respectively the vertical and horizontal eddy viscosities. The vertical component of the water velocity can be obtained from the continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0$$  \hspace{1cm} (2)

The water salinity and temperature are also calculated all over the computational domain by solving the advection–diffusion equations:

$$\frac{\partial S}{\partial t} + u \frac{\partial S}{\partial x} + v \frac{\partial S}{\partial y} + w \frac{\partial S}{\partial z} = \frac{\partial}{\partial z} \left[ K_v \frac{\partial S}{\partial z} \right] + K_h \left[ \frac{\partial^2 S}{\partial x^2} + \frac{\partial^2 S}{\partial y^2} \right]$$

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} = \frac{\partial}{\partial z} \left[ K_v \frac{\partial T}{\partial z} \right] + K_h \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} \right]$$  \hspace{1cm} (3)

where $K_h$ and $K_v$ are horizontal and vertical diffusion coefficients. The density of the water is determined from the equation of state and the turbulent viscosity is calculated by solving the turbulent kinetic energy equation [5].

All equations were solved using an explicit finite difference scheme by providing the suitable boundary conditions for seawater elevation, currents, salinity and kinetic energy. The components of the wind and bottom stresses and no flux for salinity were specified as boundary conditions at the sea surface bottom. On the other hand, along the open boundaries, a radiation condition is applied for the water velocity component normal to the boundary [6]. All the parameters in the equations have been fixed according to the corresponding values found in the literature and some of them were established by trial and error.

To solve the hydrodynamic equations, a time step of $\Delta t = 10 \text{ s}$ was adopted according to the stability criterion CFL (Current-Friederichs-Lewy). Discretization in the vertical direction consisted of dividing the water column into 35 layers with a fine resolution of 20 m for the first 30 layers and 500 m for the rest until the bottom. Boundary conditions for water fluxes were $1.25$ sverdrup (1 sverdrup = $10^3$ m$^3$/s) for the incoming flow from the Atlantic Ocean through the Strait of Gibraltar, and $1.20$ sverdrup for the outgoing flux near the bottom [7].

The solution of the 3D hydrodynamic equations provides the components of residual water currents, salinity, temperature and viscosity distributions at each
box within the calculation mesh. Fig. 3 shows the currents computed by the model at two different depths (10 m and 230 m) after a simulation of 50 d. The hydrodynamic model reproduces in a satisfactory way the overall circulation of water in the application area. The jet of Atlantic water, which enters via the Strait of Gibraltar, flows towards the Spanish coast and curves to the south to form the anticyclonic gyre (WAG). On the other hand, surface water velocity reaches its maximum values (0.55 m/s) in the Strait of Gibraltar which is in good agreement with some measured currents in the same area (0.60 m/s) [2]. Surface currents along the Spanish coast are between 0.1 and 0.4 m/s. On the other hand, the circulation of deep waters is directed towards the west with currents in the Strait of Gibraltar of about 0.15 m/s, also in good agreement with those obtained from Ref. [8]. Thus, the water dynamic is qualitatively and quantitatively reproduced by the model within the studied domain.

2.2. Dispersion of radionuclides

Transport of dissolved radionuclides in the marine environment is governed essentially by advection and diffusion processes. The 3D form of the dispersion equation that gives the time evolution of concentrations in water can be written as follows:

$$
\frac{\partial C}{\partial t} + u \frac{\partial C}{\partial x} + v \frac{\partial C}{\partial y} + w \frac{\partial C}{\partial z} =
- \frac{\partial}{\partial x} \left[ K_h \frac{\partial C}{\partial x} \right] + \frac{\partial}{\partial y} \left[ K_h \frac{\partial C}{\partial y} \right] + \frac{\partial}{\partial z} \left[ K_v \frac{\partial C}{\partial z} \right] - \lambda C
$$

(4)
where C is the radionuclide concentration in water, λ is the radioactive decay and $K_h$ and $K_v$ are, respectively, the horizontal and vertical diffusion coefficients. $K_h$ is considered the same in both directions x and y (the value taken is 2.0 m$^2$/s), while $K_v$ is always parameterized as a function of the eddy diffusivity. It is worth noting that although $^{226}$Ra is a particle reactive radionuclide [9], interactions with suspended particulate matter (SPM) have been neglected in our dispersion model because of the low SPM concentrations present in the western Alboran Sea and the Strait of Gibraltar. It has been shown experimentally that the uptake of radionuclides increases as the SPM concentration increases [10].

The advective transport is governed by the residual and tidal currents (u, v and w) previously established by the hydrodynamic models and stored in files to be used by the dispersion model. In order to reduce the numerical dispersion, the MSOU second-order scheme (Monotonic Second Order Upstream) is used to solve the advection term [11].

The model has been applied to simulate $^{226}$Ra releases from hypothetical phosphate fertilizer industries located on both the Spanish and Moroccan coasts. Fig. 4 shows an example of the time evolution in surface waters of $^{226}$Ra concentrations resulting from a single discharge at the Strait of Gibraltar. The total activity discharged is 100 kBq. After 2 d, the patch starts to be introduced in

---

**Fig. 4.** Spatiotemporal evolution in the surface water layer of $^{226}$Ra concentrations resulting from a single discharge at the Strait of Gibraltar [Grid cell (10, 27)].
the Alboran Sea due to the Atlantic jet and moves near the Spanish coast. Concentrations decrease as a consequence of dilution with uncontaminated water since the model starts from clean water. Effectively, the concentration decreased from 1000 to 0.1 Bq per volume unit in two weeks. The vertical transport of $^{226}\text{Ra}$ is also responsible for the decrease in surface concentration. The maximum vertical velocity is of the order of $10^{-4}$ m/s which makes the vertical advective transport significant only for long periods of simulation.

The patch then curves to the south and part of the $^{226}\text{Ra}$ is retained within the Alboran basin following the WAG for a considerable period, diluting progressively until reaching low concentrations at background levels. From the results above, the transport of $^{226}\text{Ra}$ in the area covered by the model is essentially governed by the water dynamics. Several numerical experiments were carried out using different conditions of $^{226}\text{Ra}$ discharges (location, duration, initial conditions) and the dispersion model has shown its consistency in producing reasonable spatiotemporal evolution of concentration within the covered domain.

REFERENCES


